Received: *October 19, 2001*, Accepted: *November 29, 2001*
Revised: *November 28, 2001*

Supersymmetric fluxbrane intersections and closed string tachyons

Jorge G. Russo
Departamento de Física, Universidad de Buenos Aires
Ciudad Universitaria, Pab. I, 1428 Buenos Aires, Argentina
E-mail: russo@df.uba.ar

Arkady A. Tseytlin*
Smith Laboratory, Ohio State University
Columbus OH 43210-1106, USA
E-mail: tseytlin.1@osu.edu

Abstract: We consider NS-NS superstring model with several "magnetic" parameters b_s $(s = 1, \ldots, N)$ associated with twists mixing a compact S^1 direction with angles in N spatial 2-planes of flat 10-dimensional space. It generalizes the Kaluza-Klein Melvin model which has single parameter b. The corresponding U-dual background is a R-R type-IIA solution describing an orthogonal intersection of N flux 7-branes. Like the Melvin model, the NS-NS string model with N continuous parameters is explicitly solvable; we present its perturbative spectrum and torus partition function explicitly for the $N = 2$ case. For generic b_s (above some critical values) there are tachyons in the S^1 winding sector. A remarkable feature of this model is that while in the Melvin $N = 1$ case all supersymmetry is broken, a fraction of it may be preserved for $N > 1$ by making a special choice of the parameters b_s. Such solvable NS-NS models may be viewed as continuous-parameter analogs of non-compact orbifold models; they and their U-dual R-R fluxbrane counterparts may have some "phenomenological" applications. In particular, in $N = 3$ case one finds a special 1/4 supersymmetric R-R 3-brane background. Putting Dp-branes in flat twisted NS-NS backgrounds leads to world-volume gauge theories with reduced amount of supersymmetry. We also discuss possible evolution patterns of unstable backgrounds towards stable supersymmetric ones.

Keywords: Superstrings and Heterotic Strings, D-branes, Conformal Field Models in String Theory.

Contents

1. Introduction

Recently, there has been a renewed interest in studying magnetic backgrounds analogous to Melvin fluxtubes (see [1]–[20]). Such backgrounds continuously interpolate between tachyonic and tachyon-free non-supersymmetric string vacua. They are also smoothly connected to the supersymmetric closed string vacua. They provide a simple framework to study the decay of unstable backgrounds by classical and quantum (non-perturbative) effects. In particular, one can compute the semiclassical amplitude for a decay of spacetime by instanton effects [5, 11] into stable supersymmetric configurations, or study the instability of type-0A string within the effective field theory and supergravity [14].

The Kaluza-Klein Melvin magnetic fluxtube background [1, 7] and the corresponding string theory [4, 6] are, in general, non-supersymmetric (except for the

*Also at Imperial College, London and Lebedev Physics Institute, Moscow.

"trivial" magnetic field values b_s, when the string theory reduces to the standard flat superstring theory). Remarkably, the direct generalization of the KK Melvin background to the case with several magnetic parameters b_s ($s = 1, \ldots, N \leq 4$), considered previously at the supergravity level in [7, 16], preserves a fraction of supersymmetry for special choices of b_s (this generalizes the observation made in [13] in the case of $N = 2$). As we shall demonstrate below, this produces a supersymmetric model which has $N - 1$ *continuous* parameters.

This NS-NS string model combines the simplicity (solvability) with supersymmetry without imposing a relation between the radius R of the KK direction and the magnetic b_s parameters. For example, the $N = 2$ model is tachyon-free for any radius only in the supersymmetric limit $b_1 = b_2$, while the 3-parameter model is supersymmetric for $b_1 = b_2 + b_3$ (modulo trivial sign changes).

The supersymmetry of special multi-parameter Melvin-type models is related to some previously known facts. In the case of the rational choice of the twist parameters $b_s R = 1/n_s$ (R is the KK radius) these models look similar [13, 16] (but are not, in fact, equivalent) to the standard supersymmetric C^N/Z_n orbifold models (with the $N = 2$ case discussed recently in [21]). For generic $b_s R$, the supersymmetric $b_1 = b_2$ case of the $N = 2$ NS-NS background is U-dual of the supersymmetric R-R type-IIA "F5-brane" background of [13] (see also [16]).

This paper is organized as follows. In section 2 we present the NS-NS superstring model, and describe the cases where a fraction of supersymmetry is preserved. In section 3 we solve the corresponding conformal string model in terms of free fields, determine the string mass spectrum and compute the one-loop partition function. As expected, the latter vanishes in the supersymmetric cases, is finite in the non-tachyonic non-supersymmetric ones, is trivial and is IR divergent in the cases where tachyons appear in the spectrum. We discuss the appearance of tachyons, and possible outcomes of evolution of the unstable backgrounds. We also consider a more general curved-space background with parameters $b_s, \tilde{b}_s$, which contains as special cases the original flat KK background and its T-dual counterpart. The corresponding non-trivial NS-NS string model is an exact conformal field theory to all orders in α', and is again solvable in terms of free fields.

In section 4 we describe U-dual supersymmetric R-R fluxbrane backgrounds, where the magnetic field (with fluxes in N different planes) corresponds to the R-R one-form potential of type-IIA supergravity. These "Fp-branes", with $p = 9 - 2N$, have proper interpretation as orthogonal intersections [7] of N F7-branes. The magnetic field parameters b_s control the strength of supersymmetry breaking, with the supersymmetric limit being $b_1 = b_2 + \cdots + b_N$, for $N = 2, 3, 4$. In particular, one finds, in addition to the "F5-brane" with 16 supersymmetries [13], an "F3-brane" with 8 supersymmetries, and an "F1-brane" with 4 supersymmetries. We argue that non-supersymmetric fluxbrane backgrounds should decay quantum mechanically via non-perturbative instanton effects into the stable supersymmetric fluxbranes.

JHEP11(2001)065

In section 5 we discuss some possible applications, and, in particular, the structure of the $\mathcal{N}$=2, $d = 4$ world-volume theory corresponding to the F3-brane. In appendix we present supergravity solutions which generalize the standard Dp-branes to the case when the flat transverse space contains magnetic twists which reduce the amount of world-volume supersymmetry.

2. The NS-NS superstring model

2.1 Definition of the model and conditions of supersymmetry

The string model we are going to consider is a direct generalization of the KK Melvin model [6], where the corresponding background [2], [5] was a flat space with one compact coordinate x_9 "mixed" with the polar angle of a spatial 2-plane. Namely, let us select N spatial 2-planes and combine the rotations in them with shifts around the KK circle [7]. The resulting metric is

$$ds_{10}^2 = -dt^2 + dx_i^2 + dy^2 + \sum_{s=1}^{N} [dr_s^2 + r_s^2(d\varphi_s + b_s dy)^2] \,, \tag{2.1}$$

where

$$y \equiv x_9 \,, \qquad i = 2N+1, \ldots, 8 \,, \qquad N = 1, 2, 3, 4 \,, \tag{2.2}$$

$$x_9 \equiv x_9 + 2\pi R \,, \qquad \varphi_s \equiv \varphi_s + 2\pi \,. \tag{2.3}$$

Though locally flat, this metric is topologically trivial only if

$$b_s R = m_s \,, \qquad m_s = 0, \pm 1, \pm 2, \ldots \,, \qquad s = 1, \ldots, N \,. \tag{2.4}$$

A similar construction can be carried out in eleven dimensions leading, upon dimensional reduction, to curved 10-d space-time geometries with R-R magnetic flux (see [7, 9, 13, 16] and section 4).

We will be interested, in particular, in the NS-NS string model defined by the locally flat 10-d metric (2.1) with $N = 2$

$$ds_{10}^2 = -dt^2 + dx_i^2 + dy^2 + dr_1^2 + r_1^2(d\varphi_1 + b_1 dy)^2 + dr_2^2 + r_2^2(d\varphi_2 + b_2 dy)^2 \,. \tag{2.5}$$

Since going around the KK circle x_9 is accompanied by the rotation in the two planes $\exp[2\pi i(b_1 R J_{12} + b_2 R J_{34})]$, the supersymmetry is trivially preserved if the integers m_1, m_2 in (2.4) are even, i.e. if $b_s R = 2n_s$. In this case, the string model is equivalent to the standard superstring in flat space (the same is obviously true for any N).

Let us now look for the cases when part of supersymmetry may be preserved. Since the space is flat (and we assume that all other supergravity fields have trivial backgrounds) the standard Killing spinor condition

$$\left(\partial_\mu + \frac{1}{4}\omega_\mu^{mn}\gamma_{mn}\right)\epsilon = 0 \tag{2.6}$$

is equivalent to the condition of existence of residual global symmetry in the Green-Schwarz string action. We shall follow closely the discussion in [6]. The light-cone gauge GS lagrangian takes a very simple form when the background geometry is flat

$$L_{GS} = L_B + L_F = G_{\mu\nu}(x)\partial_+ x^\mu \partial_- x^\nu + i\mathcal{S}_R \mathcal{D}_+ \mathcal{S}_R + i\mathcal{S}_L \mathcal{D}_- \mathcal{S}_L \,,$$
$$\mathcal{D}_a \equiv \partial_a + \frac{1}{4}\omega_\mu^{mn}\gamma_{mn}\partial_a x^\mu \,. \tag{2.7}$$

Here $\mathcal{S}^p_{R,L}$ ($p = 1, \ldots, 8$) are the right and left real spinors of SO(8) (we consider type-IIA theory).

To keep the discussion general, let us assume that the spatial part of the bosonic term in the action has the form ($y = x_9$)

$$L_B = (\partial_+ x_m - f_{mn}x_n\partial_+ y)(\partial_- x_m - f_{mk}x_k\partial_- y) + \partial_+ y\partial_- y \,, \tag{2.8}$$

where $m, n, k = 1, 2, \ldots, 2N$ and f_{mn} is a *constant* antisymmetric matrix. This may be interpreted as an action for a set of 2-d scalar fields coupled to a locally-trivial 2-d gauge potential $(A_a)_{mn} = f_{mn}\partial_a y$ taking values in the algebra of the SO($2N$) rotation group. In the simplest $N = 1$ case we have $f_{mn} = b\epsilon_{mn}$, $m, n = 1, 2$. In general, we may choose the coordinates so that f_{mn} takes a block-diagonal form:

$$f_{s,s+1} = -f_{s+1,s} = b_s \,, \qquad s = 1, 2, \ldots, N \,.$$

N (which may be 1, 2, 3 or 4) is the number of 2-planes where f_{mn} has non-zero components.

In the natural vierbein basis ($e_m = dx_m - f_{mn}x_n dy$, $e_y = dy$) the spin connection 1-form is $\omega^{mn} = -f_{mn}dy$, $\omega^{m,y} = 0$ so that the fermionic part of (2.7) becomes

$$L_F = i\mathcal{S}_R\left(\partial_+ - \frac{1}{4}f_{mn}\gamma_{mn}\partial_+ y\right)\mathcal{S}_R + i\mathcal{S}_L\left(\partial_- - \frac{1}{4}f_{mn}\gamma_{mn}\partial_- y\right)\mathcal{S}_L \,. \tag{2.9}$$

To look for a residual global supersymmetry $\mathcal{S} \to \mathcal{S} + \epsilon$ we are thus to solve the zero-mode (Killing-spinor) equation (2.6) for a space-time spinor $\epsilon = \epsilon(x^i, y)$:

$$\left(\partial_y - \frac{1}{4}f_{mn}\gamma_{mn}\right)\epsilon = 0 \,. \tag{2.10}$$

The formal solution is

$$\epsilon(y) = \exp\left(\frac{1}{4}f_{mn}\gamma_{mn}y\right)\epsilon_0 \,, \qquad \epsilon_0 = \text{const} \,. \tag{2.11}$$

It does not, in general, satisfy the necessary periodic boundary condition in y, $\epsilon(y + 2\pi R) = \epsilon(y)$, unless f_{mn} is such that

$$e^{\frac{1}{2}\pi R f_{mn}\gamma_{mn}}\epsilon_0 = \epsilon_0 \tag{2.12}$$

JHEP11(2001)065

has non-trivial solutions for ϵ_0. In the simplest $N = 1$ case, when f_{mn} has just one non-zero eigen-value $f_{12} = b$, the condition (2.12) becomes $\exp(\pi b R \gamma_{12})\epsilon_0 = [\cos(\pi b R) + \gamma_{12}\sin(\pi b R)]\epsilon_0$, so that the supersymmetry is preserved only if $bR = 2n$. In this case the model becomes trivial, i.e. equivalent to the flat space superstring.

In the case of $N \geq 2$ non-zero eigenvalues of f_{mn} the full supersymmetry is again trivially preserved if $b_s R = 2n_s$ $(s = 1, \ldots, N)$.[1] But now there is also a possibility of preserving a fraction of supersymmetry without relating R to b_s but instead imposing a condition on b_s and choosing a special solution for ϵ_0. Indeed, let us look for solutions of

$$f_{mn}\gamma_{mn}\epsilon_0 = 0\,. \tag{2.13}$$

This problem is isomorphic to the condition of preservation of a fraction of supersymmetry in (abelian) gauge field theory in $2N$ dimensions in a magnetic gauge field background, i.e. to the condition of vanishing of the gaugino variation. In the case of two non-zero eigenvalues b_1, b_2 one finds that 1/2 of supersymmetry is preserved if the configuration is selfdual $b_1 = b_2$ (or anti-selfdual $b_1 = -b_2$). Indeed, in this case we get from (2.13)

$$(\gamma_{12} + \gamma_{34})\epsilon_0 = 0\,, \tag{2.14}$$

which is a projector relation

$$P_-\epsilon_0 = 0\,, \qquad P_+\epsilon_0 = \epsilon_0\,, \qquad P_\pm = \frac{1}{2}(1 \pm \gamma_{1234})\,, \qquad P_+ + P_- = 1\,. \tag{2.15}$$

More generally, if we assume that ϵ_0 solves (2.14) then the periodicity condition (2.12) becomes

$$e^{\pi R(b_1 - b_2)\gamma_{12}}\epsilon_0 = \epsilon_0\,, \tag{2.16}$$

so that supersymmetry — a y-periodic Killing spinor — exists if

$$R(b_1 - b_2) = 2n = 0, \pm 2, \pm 4, \ldots\,. \tag{2.17}$$

Under the assumption $0 < Rb_s < 2$ that leaves us with $R(b_1 - b_2) = 0$, i.e. $b_1 = b_2$ as the only *non-trivial* option. In this case 1/2 of 32 original supersymmetries is preserved.

For three non-zero eigen-values (twists in three planes) we get from (2.13) the following condition (multiplying by γ_{12})

$$\left(1 - \frac{b_2}{b_1}\gamma_{1234} - \frac{b_3}{b_1}\gamma_{1256}\right)\epsilon_0 = 0\,. \tag{2.18}$$

While the operator here is never a projector, this equation can be satisfied if we set, e.g.,

$$b_1 = b_2 + b_3\,, \tag{2.19}$$

[1] The parameters b_s are, in fact, defined modulo shifts by $2n_s/R$, so non-trivial string models can be parametrized, e.g., by $0 < b_s < 2/R$. We shall assume this in what follows.

JHEP11(2001)065

and impose the following two conditions (two copies of the "self-duality" conditions that are realized by projectors)

$$(\gamma_{12}+\gamma_{34})\epsilon_0 = 0\,, \qquad (\gamma_{12}+\gamma_{56})\epsilon_0 = 0\,. \tag{2.20}$$

Other possibilities are obviously equivalent to this one by changing signs and renaming the parameters. The resulting solution preserves 1/4 of supersymmetry.[2] More general solutions are obtained by imposing (2.20) and then solving the periodicity condition (2.12): $e^{\pi R(b_1-b_2-b_3)\gamma_{12}}\,\epsilon_0 = \epsilon_0$, i.e.

$$R(b_1-b_2-b_3) = 2n = 0, \pm 2, \pm 4, \ldots \tag{2.21}$$

By virtue of the periodicity in b_s ($b_s \equiv b_s + 2n_s R^{-1}$), the resulting theory is equivalent to the one with b_s related by (2.19).

Let us note that for b_s in the "fundamental domain" $0 < Rb_s < 2$, the condition (2.21) is formally solved not only by (2.19), but also by

$$R(b_1-b_2-b_3) = -2\,. \tag{2.22}$$

While the resulting string theory is equivalent, as mentioned above, to the one defined by (2.19), here the Killing spinor has a non-trivial dependence on y, i.e. is not constant along the KK direction. This implies that upon dimensional reduction in y or T-duality in y the supersymmetry will not be preserved at the *supergravity* level (but will of course be preserved at the level of the perturbative *string* theory).[3]

We conclude that in the case of the two and more twist parameters it is possible to preserve a fraction of supersymmetry without imposing a relation between the KK radius R and the twists. The resulting non-compact NS-NS string models with continuous parameters provide a simple laboratory for study of issues of supersymmetry breaking and tachyons in closed string theory.

In the special case of rational parameters, e.g., $b_s R = 1/n_s$, these models look similar [3, 13, 16] to the non-compact orbifold models or strings on the cone [24]. Indeed, in the $N=1$ case the coordinate $\varphi' = \varphi + by$ in (2.1) becomes $2\pi/n$ periodic. Introducing the new 2π-periodic coordinates $\tilde\varphi = n\varphi'$ and the new $2\pi R$ periodic coordinate $\tilde y = R\varphi$ one finds that the metric becomes

$$ds^2_{10} = -dt^2 + dx_i^2 + n^2\left(d\tilde y - \frac{1}{n}d\tilde\varphi\right)^2 + dr^2 + \frac{r^2}{n^2}d\tilde\varphi^2\,. \tag{2.23}$$

[2] Related gauge field configuration appears in SYM theory on a 6-torus (see e.g. [22]).

[3] In the case of reduction from 11 to 10 dimensions one again will have loss of supersymmetry at the level of supergravity (and, in this case, also at the level of perturbative string theory in the corresponding R-R background). However, supersymmetry which is present in the full 11-d (membrane) theory should be restored in the 10-d string theory once non-perturbative string states are taken into account (see [23] for a related discussion).

JHEP11(2001)065

The 2-plane part of this metric is indeed a metric of a cone, but in addition there is a non-vanishing KK vector. Thus the metric cannot be represented as a direct product of a cone and a circle, and its non-trivial 3-d part is, in fact, *non*-singular (the metric (2.1) has no conical singularities for any $b_s R$). As a result, the corresponding string model (in particular, its spectrum) is not equivalent to the one for the direct product of an orbifold C/Z_n and a circle S^1. There is, however, a close similarity, in particular regarding instabilities and the presence of supersymmetry for $N > 1$ for a special choice of n_s. For example, in the $N = 2$ case the $\mathrm{C}^2/(Z_{n_1} \times Z_{n_2})$ orbifold is also supersymmetric if $n_1 = \pm n_2$, i.e. $b_1 = \pm b_2$ [21, 13].

3. Solution of the (b_1, b_2) NS-NS string model

3.1 Free field representation

It is possible to express the bosonic x^m and fermionic $\mathcal{S}_{L,R}$ coordinates in (2.8), (2.9) terms of free fields. For simplicity, we shall explicitly consider the case of the two non-zero eigenvalues $b_1 = f_{12}$, $b_2 = f_{34}$. The Lorentz group SO(8) is then broken to:

$$\mathrm{SO}(8) \to \mathrm{SO}(4) \times \mathrm{SO}(2) \times \mathrm{SO}(2)\,,$$

where the two factors SO(2) represent rotations in the 1-2 and 3-4 plane, respectively. Fermion representations decompose as follows:

$$\mathcal{S}_R = \psi_R^{++} \oplus \bar\psi_R^{--} \oplus \psi_R^{+-} \oplus \bar\psi_R^{-+}\,,$$
$$8_R \to \left(2_R, \frac{1}{2}, \frac{1}{2}\right) \oplus \left(\bar 2_R, -\frac{1}{2}, -\frac{1}{2}\right) \oplus \left(2_R, \frac{1}{2}, -\frac{1}{2}\right) \oplus \left(\bar 2_R, -\frac{1}{2}, \frac{1}{2}\right)$$

and the same for $\mathcal{S}_L$. The bosonic and fermionic parts of the GS lagrangian (2.8), (2.9) take the form:

$$\begin{aligned} L_B = {} & \partial_+ x_i \partial_- x_i + (\partial_+ + i b_1 \partial_+ y) z_1\, (\partial_- - i b_1 \partial_- y) z_1^* + \\ & + (\partial_+ + i b_2 \partial_+ y) z_2\, (\partial_- - i b_2 \partial_- y) z_2^* + \partial_+ y \partial_- y\,, \\ & z_1 \equiv x_1 + i x_2\,, \qquad z_2 \equiv x_3 + i x_4\,, \end{aligned}$$

and

$$\begin{aligned} L_F = {} & i\bar\psi_R^{--}\left[\partial_+ + \frac{1}{2} i (b_1 + b_2)\partial_+ y\right]\psi_R^{++} + i\bar\psi_R^{-+}\left[\partial_+ + \frac{1}{2} i (b_1 - b_2)\partial_+ y\right]\psi_R^{+-} + \\ & + i\bar\psi_L^{--}\left[\partial_- + \frac{1}{2} i (b_1 + b_2)\partial_- y\right]\psi_L^{++} + i\bar\psi_L^{-+}\left[\partial_- + \frac{1}{2} i (b_1 - b_2)\partial_- y\right]\psi_L^{+-}\,. \end{aligned} \tag{3.1}$$

The lagrangian can be written in terms of redefined bosons and fermions which are free fields

$$\begin{aligned} z_1 &= e^{i b_1 y} Z_1\,, & z_2 &= e^{i b_2 y} Z_2\,, \\ Z_1(\sigma + \pi) &= e^{2 i \pi b_1 w R} Z_1(\sigma)\,, & Z_2(\sigma + \pi) &= e^{2 i \pi b_2 w R} Z_2(\sigma)\,, \\ \psi_R^{++} &= e^{-\frac{i}{2}(b_1 + b_2) y} \psi_{0R}^{++}\,, & \psi_{0R}^{++}(\sigma + \pi) &= e^{i \pi w R (b_1 + b_2)} \psi_{0R}^{++}(\sigma)\,, \quad \text{etc.}\,, \end{aligned} \tag{3.2}$$

where w is the winding number in the y direction.

From these equations it is easy to see why the case of $b_1 = b_2$ (or $b_1 = -b_2$) is supersymmetric. Then the fermions $\psi^{+-}_{R,L}$ and $\bar{\psi}^{-+}_{R,L}$ are decoupled from y, i.e. are free fields that do not transform under $y \to y + 2\pi R$. There are four free Weyl fermions with nontrivial periodicity: ψ^{++}_{0L}, ψ^{++}_{0R}, $\bar{\psi}^{--}_{0L}$, $\bar{\psi}^{--}_{0R}$. But they have the same boundary conditions as the bosonic degrees of freedom $Z_{1L}, Z_{1R}, Z_{2L}, Z_{2R}$. Thus, for every fermionic degree of freedom, there is a bosonic degree of freedom with the same periodicity. This 2-d supersymmetry is correlated with the space-time one in the light-cone gauge GS description.

An interesting special case is when $b_1 = b_2 = R^{-1}(n + 1/2)$. Here there are free fermions obeying antiperiodic boundary conditions in $y = x_9$ in the odd winding number sector, but at the same time the supersymmetry is preserved since there are also antiperiodic free bosons Z_1, Z_2.[4] Note that for a generic value of $b_1 = b_2$ this model is not equivalent to the standard (periodic or antiperiodic) free superstring theory in flat space. The non-triviality of the model expressed in terms of the free fields is caused by the special boundary conditions.[5]

3.2 Mass spectrum

This superstring model can be solved by a simple generalization of the discussion in the case of $b_1 \neq 0, b_2 = 0$ in [6]. Let $\hat{N}_R$ and $\hat{N}_L$ denote the number of states operators, which have the same form as in the free superstring theory. They have integer non-negative eigenvalues in the GS description, while in the NSR approach they are expressed in terms of normal-ordered operators as

$$\hat{N}_{R,L} = N_{R,L} - a\,, \qquad a^{(\mathrm{R})} = 0\,, \qquad a^{(\mathrm{NS})} = \frac{1}{2}\,. \tag{3.3}$$

Let us introduce the angular momentum operators $\hat{J}_1 \equiv \hat{J}_{12}$ and $\hat{J}_2 \equiv \hat{J}_{34}$, which generate rotations in the respective 2-planes (shifts in φ_1 and φ_2 in (2.5)). They can be written as

$$\hat{J}_s = \hat{J}_{sL} + \hat{J}_{sR}\,, \quad \hat{J}_{sL} = l_{sL} + \frac{1}{2} + S_{sL}\,, \quad \hat{J}_{sR} = -l_{sR} - \frac{1}{2} + S_{sR}\,, \qquad s = 1, 2\,, \tag{3.4}$$

where the orbital momenta in each plane $l_{L,R} = 0, 1, 2, \ldots$ are related to the Landau quantum number l and the radial quantum number k by $l = l_L - l_R$ and $2k = l_L + l_R - |l|$, and $S_{sR,L}$ are the spin components. In the NS-NS sector, their possible values satisfy the condition

$$|S_{1R} \pm S_{2R}| \leq \hat{N}_R + 1\,, \qquad |S_{1L} \pm S_{2L}| \leq \hat{N}_L + 1\,.$$

[4]The physical fermion and boson coordinates in (3.1) and (3.1) do not of course transform under shifts of x_9.

[5]This class of models is closely related to the to the Scherk-Schwarz type compactifications in string theory [25, 26].

JHEP11(2001)065

The mass spectrum is given by (cf. [6])

$$\alpha' M^2 = 2(\hat{N}_R + \hat{N}_L) + \frac{\alpha'}{R^2}(m - b_1 R\hat{J}_1 - b_2 R\hat{J}_2)^2 + \\ + \frac{R^2 w^2}{\alpha'} - 2\hat{\gamma}_1(\hat{J}_{1R} - \hat{J}_{1L}) - 2\hat{\gamma}_2(\hat{J}_{2R} - \hat{J}_{2L})\,, \tag{3.5}$$

where

$$\hat{N}_R - \hat{N}_L = mw\,, \quad \hat{\gamma} \equiv \gamma - [\gamma]\,, \quad \gamma_1 \equiv b_1 R w\,, \qquad \gamma_2 \equiv b_2 R w\,. \tag{3.6}$$

$[\gamma]$ denotes the integer part of γ (so that $0 \le \hat{\gamma} < 1$).

In a generic magnetic background, one expects that all supersymmetries will be broken since fermions and bosons get different mass shifts: the gyromagnetic interaction is proportional to $b_s J_s$, which is indeed different for fermions and bosons. The supersymmetry in the case of $b_1 = b_2$ is due to a compensation between the two gyromagnetic contributions corresponding to the two 2-planes. Let us consider, as an illustration, the fermion and boson states with the winding number $w = 0$, K-K charge m, and $\hat{J}^F_{1,2} = 1/2$, $\hat{J}^B_1 = 1$, $\hat{J}^B_2 = 0$. They have the same mass at zero magnetic parameters $b_1 = b_2 = 0$, and there is a mass splitting for generic b_1, b_2, proportional to

$$\delta M^2 = (b_1 - b_2)\left[\frac{m}{R} - \frac{1}{4}(3b_1 + b_2)\right].$$

It vanishes in the supersymmetric case $b_1 = b_2$. As discussed above, the case $b_1 = -b_2$ is also supersymmetric (giving equivalent CFT related by the redefinition $\varphi_2 \to -\varphi_2$). In this case, the superpartner of the fermion with $\hat{J}^F_{1,2} = 1/2$ is the boson with $\hat{J}^B_1 = -1$, $\hat{J}^B_2 = 0$.

3.3 Partition function

It is easy to compute the partition function in the GS formulation following the discussion in [6]. We first expand y in eigenvalues of the laplacian on the 2-torus and redefine the fields z_1, z_1^*, z_2, z_2^* and $\psi^{+\pm}_{R,L}, \bar{\psi}^{-\mp}_{R,L}$ to eliminate the non-zero-mode part of y from the U(1) connection. The zero-mode part of y on the torus ($ds^2 = |d\sigma_1 + \tau d\sigma_2|^2$, $\tau = \tau_1 + i\tau_2$, $0 < \sigma_a \le 1$) is $y_* = y_0 + 2\pi R(w\sigma_1 + w'\sigma_2)$, where w, w' are integer winding numbers. Integrating over the bosonic anf fermionic fields we get a ratio of determinants of scalar operators of the type $\partial + iA$, $\bar{\partial} - i\bar{A}$ ($\partial = \frac{1}{2}(\partial_2 - \tau\partial_1)$) with constant connections

$$A_s = b_s \partial y_* = \pi \chi_s\,, \qquad \bar{A}_s = b_s \bar{\partial} y_* = \pi \bar{\chi}_s\,, \\ chi_s \equiv b_s R(w' - \tau w), \qquad \bar{\chi}_s \equiv b_s R(w' - \bar{\tau} w)\,, \qquad s = 1, 2\,. \tag{3.7}$$

The final expression for the partition function takes the following simple form

$$Z(R, b_1, b_2) = cV_5 R \int \frac{d^2\tau}{\tau_2^2} \sum_{w,w'=-\infty}^{\infty} \exp\left(-\frac{\pi}{\alpha'\tau_2} R^2 |w' - \tau w|^2\right) \mathcal{Z}_0(\tau, \bar{\tau}; \chi_s, \bar{\chi}_s) \times \tag{3.8} \\ \times \frac{Y^2(\tau, \bar{\tau}; \frac{1}{2}(\chi_1 + \chi_2), \frac{1}{2}(\bar{\chi}_1 + \bar{\chi}_2)) Y^2(\tau, \bar{\tau}; \frac{1}{2}(\chi_1 - \chi_2), \frac{1}{2}(\bar{\chi}_1 - \bar{\chi}_2))}{Y(\tau, \bar{\tau}; \chi_1, \bar{\chi}_1) Y(\tau, \bar{\tau}; \chi_2, \bar{\chi}_2)}\,.$$

Here

$$Y(\tau,\bar\tau;\chi,\bar\chi) \equiv \frac{\det{}'(\partial+i\pi\chi)\ \det{}'(\bar\partial-i\pi\bar\chi)}{\det{}'\partial\ \det{}'\bar\partial} = \frac{U(\tau,\bar\tau;\chi,\bar\chi)}{U(\tau,\bar\tau;0,0)}\,, \tag{3.9}$$

$$U(\tau,\bar\tau;\chi,\bar\chi) \equiv \prod_{(n,n')\neq(0,0)} (n'-\tau n+\chi)(n'-\bar\tau n+\bar\chi)\,, \tag{3.10}$$

where, in the determinants, we have projected out the zero modes appearing at $\chi=\bar\chi=0$ (i.e. $Y(\tau,\bar\tau;0,0)=1$). The equivalent form of Y is

$$Y(\tau,\bar\tau;\chi,\bar\chi) = \exp\left[\frac{\pi(\chi-\bar\chi)^2}{2\tau_2}\right]\frac{\theta_1(\chi|\tau)}{\chi\theta_1'(0|\tau)}\frac{\theta_1(\bar\chi|\bar\tau)}{\bar\chi\theta_1'(0|\bar\tau)} = \left|\frac{\theta[\binom{1/2+bRw}{1/2+bRw'}](0|\tau)}{bR(w'-\tau w)\theta_1'(0|\tau)}\right|^2, \tag{3.11}$$

where $\theta_1(\chi|\tau)=\theta[\binom{1/2}{1/2}](\chi|\tau)$. In particular,

$$Y\left(\tau,\bar\tau;\frac{1}{2}(\chi_1+\chi_2),\frac{1}{2}(\bar\chi_1+\bar\chi_2)\right) = \left|\frac{\theta[\binom{1/2+1/2(b_1+b_2)Rw}{1/2+1/2(b_1+b_2)Rw'}](0|\tau)}{\frac{1}{2}(b_1+b_2)R(w'-\tau w)\theta_1'(0|\tau)}\right|^2. \tag{3.12}$$

The factor $\mathcal{Z}_0$ in (3.8) stands for the contributions of the integrals over the constant parts of the fields $z_{1,2}$, $z^*_{1,2}$ and $\psi_{R,L}$, $\bar\psi_{R,L}$ (i.e. the contributions of $(n,n')=(0,0)$ terms in the determinants)

$$\mathcal{Z}_0 = \tau_2^{-2}2^{-8}R^4|w'-\tau w|^4\frac{(b_1+b_2)^4(b_1-b_2)^4}{b_1^2b_2^2}\,. \tag{3.13}$$

Note that the full integrand of Z is modular invariant since the transformation of τ can be combined with a redefinition of w,w' (so that, e.g., $\mathcal{Z}_0$ and Y remain invariant).

As expected, the partition function vanishes in the supersymmetric limit $b_1=b_2$ or $b_1=-b_2$. More generally, due to the periodicity in b_1R and b_2R,

$$Z(R,b_1,b_2) = Z(R,b_1+2n_1R^{-1},b_2+2n_2R^{-1})\,, \qquad n_1,n_2=0,\pm1,\ldots\,, \tag{3.14}$$

it vanishes also at $b_1\pm b_2=2nR^{-1}$ (these points are zeroes of the theta-functions appearing in the numerator of (3.8)). The divergence at $b_{1,2}\to0$ (see eq. (3.13)) corresponds to the restoration of translational invariance in the 1-2 and 3-4 planes in the zero magnetic field limit. This divergence reproduces the factors of areas of the 2-planes. If both $b_1,b_2\to0$, the divergence is cancelled against the fermion factors in the numerators, so that Z also vanishes, as it should in the supersymmetric zero-field limit.

Z is infrared-divergent for those values of the parameters b_1,b_2 and R for which there are tachyonic states in the spectrum. This is seen by Poisson resummation in w' and expansion of the integrand of (3.8) at large τ_2 as in [4]. The integral over τ_2 in each term in the sum diverges at large τ_2 in the presence of tachyons, and is finite when tachyons are absent.

3.4 Tachyonic states, T-dual background and its possible evolution

Let us discuss the tachyonic states in this model for non-vanishing b_1, b_2. Without loss of generality, we can assume $b_1 \geq b_2 > 0$. As in [6], the state with lowest value for M^2 is a particular spin 2 winding state with

$$\hat{N}_R = \hat{N}_L = 0\,, \qquad w = 1\,, \qquad m = 0\,, \qquad l_{1L,R} = l_{2L,R} = 0\,, \\ S_{1R} = -S_{1L} = 1\,, \qquad S_{2R} = S_{2L} = 0\,, \tag{3.15}$$

so that $\hat{J}_1 = \hat{J}_2 = 0$, $\hat{J}_{1R} - \hat{J}_{1L} = 1$, $\hat{J}_{2R} - \hat{J}_{2L} = -1$. Its mass (see (3.5))

$$M^2 = \frac{R^2}{\alpha'^2} - \frac{2R}{\alpha'}(b_1 - b_2) \tag{3.16}$$

becomes tachyonic for

$$b_1 - b_2 > \frac{R}{2\alpha'}\,. \tag{3.17}$$

The negative contribution to M^2 originates from the gyromagnetic interactions in (3.5) depending on non-zero winding number.[6]

The theory with $b_1 = b_2$ (or $b_1 = -b_2$) is tachyon-free for any radius R. This is in agreement with the above discussion implying the presence of partial unbroken supersymmetry for $b_1 = \pm b_2$.

The non-supersymmetric string models with $R \geq 2\alpha'(b_1 - b_2)$ are also tachyon-free. It is easy to show that the integral over τ_2 in the one-loop partition function (3.8) is finite in this case, for each term in the sum.

An important question is how the string background reacts to the presence of tachyons. To address this problem, it is useful to consider the T-dual string model, where the tachyon has the same quantum numbers as in (3.15), except for interchanged winding and momentum numbers, i.e. $w = 0$ and $m = 1$. Since the tachyon (3.15) now appears not in the winding but in the *momentum* sector, it is a state of the supergravity multiplet. That means that, as in the 1-parameter case discussed in [14], here the instability can be seen directly at the level of the supergravity equations expanded near the T-dual background. Above some critical radius, the space-time background becomes unstable under the tachyonic perturbations of the supergravity fields.

[6] As follows from the discussion of the solution of the tachyon equation in [4, 6, 14] the tachyon wave function in the $N = 1$ Melvin model is localized (with finite width related to $\gamma = bRw$) near $r = 0$. The same will be true for $N > 1$ models. This is similar to localization of tachyons at fixed planes in orbifold models. Another analogy is that here the tachyonic states must have non-zero S^1 winding number *and* spin in a 2-plane while the tachyons in orbifold models appear in the twisted sector.

JHEP11(2001)065

Applying T-duality in the $y = x_9$ direction to (2.5) gives the following T-dual background (string-frame metric, dilaton, and NS-NS 2-form)[7]

$$\begin{aligned} ds^2_{10} &= -dt^2 + dx_i^2 + dr_1^2 + dr_2^2 + r_1^2 d\varphi_1^2 + r_2^2 d\varphi_2^2 + \\ &\quad + f^{-1}[dy^2 - (\tilde{b}_1 r_1^2 d\varphi_1 + \tilde{b}_2 r_2^2 d\varphi_2)^2] \,, \end{aligned} \tag{3.18}$$

$$e^{2(\phi-\phi_0)} = f^{-1} \,, \qquad B_2 = f^{-1}(\tilde{b}_1 r_1^2 d\varphi_1 + \tilde{b}_2 r_2^2 d\varphi_2) \wedge dy \,, \tag{3.19}$$

$$f \equiv 1 + \tilde{b}_1^2 r_1^2 + \tilde{b}_2^2 r_2^2 \,. \tag{3.20}$$

Here, for convenience, we have renamed $(b_1, b_2) \to (\tilde{b}_1, \tilde{b}_2)$. The counterpart of the state (3.16) has the mass ($R \to \tilde{R} = \alpha'/R$)

$$M^2 = \frac{1}{R^2} - \frac{2}{R}(\tilde{b}_1 - \tilde{b}_2) \,. \tag{3.21}$$

Thus this background is unstable for $R > R_{\rm cr}$, $R_{\rm cr} = (2\tilde{b}_1 - 2\tilde{b}_2)^{-1}$.

Consider now the case in which the magnetic field is $\tilde{b}_1 = \tilde{b}_2 + 1/2R + \epsilon$, $\epsilon > 0$. For small ϵ, the only tachyonic state is the first Kaluza-Klein mode with $m = 1$ and mass (3.21). The corresponding perturbation of the background (3.18), (3.19), as a solution of the linearized supergravity equations, will then grow exponentially with time. The explicit form of this tachyonic perturbation mode gives an indication (at least to linear order) of the evolution of the geometry from the unstable state. Having $m = 1$ and vanishing orbital angular momenta (see (3.15)), the spatial dependence of the unstable mode is of the form $f(r_1, r_2) \cos\frac{y}{R}$. Since the tachyonic state has $S_{1R}, S_{1L} \neq 0$, the perturbation will modify the metric and the antisymmetric tensor in their parts corresponding to the 2-plane (r_1, φ_1).

This suggests that the background (3.18), (3.19) will be evolving towards a new solution which is *not* translational invariant in $y = x_9$. There is some similarity with the fate of the well-known instability [27] of the black string solution. The latter background (which, like (3.18), has translational isometry in x_9) becomes unstable for some $R > R_0$, and seems to evolve into a configuration which is no longer translationally invariant in x_9 [28].

To try to check this conjecture, one is to generalize (3.18), (3.19) to include time dependence (in particular, making the effective $\tilde{b}_s$ parameters "run" with time). It would be interesting to find the explicit solution for the end-point of the evolution. The resulting background should be stable at the supergravity level, but may still be unstable at the full quantum string theory level.

In general, non-supersymmetric string models that are stable (non-tachyonic) at the classical level may become unstable at the quantum string level. For example, the tachyon-free string background with $R \geq 2\alpha'(b_1 - b_2)$, $b_1 \neq \pm b_2$ may decay due to quantum-mechanical effects. Since the physical mechanism for the decay here is different from the one discussed above the final state of the evolution may also be

[7]Note that this solution has regular curvature and the dilaton can be made small everywhere.

different. If initially b_1 is very close to b_2, then it is natural to expect that this model will evolve towards a (quantum-mechanically) stable supersymmetric model with $b_1' = b_2' < \min(b_1, b_2)$ (see related discussion in section 4).[8]

3.5 Four-parameter generalization

As in the $N = 1$ case [6], the T-dual string model based on (3.18), (3.19) admits a straightforward generalization: introducing two extra parameters into (3.18), (3.19) by the locally trivial transformation $\varphi_s \to \varphi_s + b_s x_9$ we get a string model that depends on the four magnetic parameters $(b_1, b_2, \tilde{b}_1, \tilde{b}_2)$ (as well as R). The corresponding σ-model lagrangian is then given by

$$\begin{aligned} L = {} & \partial_+ x_i \partial_- x_i + \partial_+ r_1 \partial_- r_1 + \partial_+ r_2 \partial_- r_2 + r_1^2 \partial_+ \varphi_1' \partial_- \varphi_1' + r_2^2 \partial_+ \varphi_2' \partial_- \varphi_2' + \\ & + f^{-1} \Big[\partial_+ y \partial_- y - (\tilde{b}_1 r_1^2 \partial_+ \varphi_1' + \tilde{b}_2 r_2^2 \partial_+ \varphi_2')(\tilde{b}_1 r_1^2 \partial_- \varphi_1' + \tilde{b}_2 r_2^2 \partial_- \varphi_2') \Big] + \\ & + f^{-1} \Big[\tilde{b}_1 r_1^2 (\partial_+ \varphi_1' \partial_- y - \partial_- \varphi_1' \partial_+ y) + \tilde{b}_2 r_2^2 (\partial_+ \varphi_2' \partial_- y - \partial_- \varphi_2' \partial_+ y) \Big] + \\ & + \mathcal{R} \left(\phi_0 - \frac{1}{2} \ln f \right) , \end{aligned} \tag{3.22}$$

$$\varphi_1' \equiv \varphi_1 + b_1 y \, , \qquad \varphi_2' \equiv \varphi_2 + b_2 y \, , \qquad y = x_9 \equiv x_9 + 2\pi R \, . \tag{3.23}$$

This four-parameter model is exactly solvable, despite its complicated curved-space geometry. The computation of its spectrum can be done by a simple generalization of the case $b_2 = \tilde{b}_2 = 0$ considered in [6] and the $\tilde{b}_1 = \tilde{b}_2 = 0$ case in (3.5)–(3.6). One finds (cf. (3.5))

$$\begin{aligned} \alpha' M^2 = {} & 2(\hat{N}_R + \hat{N}_L) + \frac{\alpha'}{R^2}(m - b_1 R \hat{J}_1 - b_2 R \hat{J}_2)^2 + \\ & + \frac{\alpha'}{\tilde{R}^2}(w - \tilde{b}_1 \tilde{R} \hat{J}_1 - \tilde{b}_2 \tilde{R} \hat{J}_2)^2 - 2\hat{\gamma}_1 (\hat{J}_{1R} - \hat{J}_{1L}) - 2\hat{\gamma}_2 (\hat{J}_{2R} - \hat{J}_{2L}) \, , \\ & \hat{N}_R - \hat{N}_L = mw \, , \qquad \tilde{R} = \frac{\alpha'}{R} \, , \qquad \hat{\gamma}_s \equiv \gamma_s - [\gamma_s] \, , \\ & \gamma_1 \equiv b_1 R w + \tilde{b}_1 \tilde{R} m - \alpha' \tilde{b}_1 b_s \hat{J}_s \, , \qquad \gamma_2 \equiv b_2 R w + \tilde{b}_2 \tilde{R} m - \alpha' \tilde{b}_2 b_s \hat{J}_s \, . \end{aligned} \tag{3.24}$$

It is possible also to compute the partition function, following the methods of [6].[9] A close inspection of the spectrum shows that it is supersymmetric in the case when $b_1 = b_2$, $\tilde{b}_1 = \tilde{b}_2$ (or $b_1 = -b_2$, $\tilde{b}_1 = -\tilde{b}_2$). Special cases are $\tilde{b}_1 = \tilde{b}_2 = 0$ or $b_1 = b_2 = 0$ when this model reduces to the $N = 2$ Melvin model with $b_1 = \pm b_2$ or to its T-dual (3.18), (3.19) with $\tilde{b}_1 = \pm \tilde{b}_2$.

[8] In the special case of rational $b_s R = 1/n_s$, when the model becomes similar to the orbifold model, this is analogous to the picture suggested in [21]: a non-supersymmetric $n_1 \neq \pm n_2$ unstable C^2/Z_m orbifold should decay into the supersymmetric one (see also [20]).

[9] This was recently done in [20] which appeared after the completion of the present work.

4. Supersymmetric R-R vector fluxbrane backgrounds

4.1 Supersymmetry

Starting with the background (2.1) (or (2.5) and its T-dual (3.18), (3.19)) we may now construct other related solutions by applying U-duality. An equivalent procedure is to start with the same flat background in $d = 11$, with the KK coordinate y now interpreted as x_{11}, and obtain type-IIA magnetic R-R flux tube backgrounds by reducing along x_{11}. This will give an N-parameter generalization of the R-R flux 7-brane [7, 9].

Let us first discuss the issue of supersymmetry of the resulting solutions on the example of the 2-parameter case. The background described by the locally flat metric (2.5) is obviously 1/2 supersymmetric for $b_1 = b_2$ as a solution of 11-d supergravity: the condition of existence of Killing spinors in $d = 11$ is the same (2.11)–(2.12) as in $d = 10$. The supersymmetry should of course be preserved at the full M-theory (membrane theory) level, but it might be broken by the $11 \to 10$ reduction if one restricts consideration to the 10-d supergravity (or perturbative 10-d string theory) states only. The necessary condition for preserving the supersymmetry at the supergravity level is that Killing spinors should be constant along the direction of compactification.[10] In other words, the supercharges that are preserved by the reduction form a subset of those of the original solution that commute with translations along the KK direction.

As discussed in section 2, in the case of $b_1 = b_2$ in $N = 2$ model (and in similar $N > 2$ cases like (2.19)) the Killing spinors are actually constant, i.e. do not depend on y. That means that all of the 16 supersymmetries of the 11-d solution must be present in the resulting type-IIA solution.[11] From the 10-d supergravity perspective, the supersymmetry condition $b_1 = \pm b_2$ should arise from solving the type-IIA Killing spinor equation in the corresponding curved R-R background.

This is no longer so in other possible 11-d supersymmetric cases (like (2.22)) where there is a relation between the b_s-parameters and R: there the Killing spinor depends on y and the supersymmetry will be broken at the supergravity level by the dimensional reduction. The simplest example is provided by the special 1-parameter case $b_1 R = 2$, $b_2 = 0$ [9]. Here the 11-d space is globally flat ($\varphi_1' = \varphi_1 + by$ is a globally defined 2π-periodic coordinate), i.e. the 11-d theory is equivalent to the flat

[10]More precisely, the number of supersymmetries preserved upon reduction of a supersymmetric solution is equal to the number of Killing spinors which have vanishing Lie derivative along the direction of reduction (see [29] and references there for a discussion and examples).

[11]It should be straightforward to construct the GS string action in the corresponding R-R background in IIA theory by using the same method as in [9], i.e. by starting with the corresponding supermembrane action in flat 11-d background. The existence of a residual global supersymmetry of the resulting GS action would then imply supersymmetry and thus stability of the magnetic R-R fluxbrane background.

M-theory with 32 supersymmetries. However, the resulting 10-d background is non-trivial and non-supersymmetric (both at the level of type-IIA supergravity and the perturbative string theory in the corresponding R-R background [9]). It is only after the non-perturbative (wound membrane) M-theory states are taken into account, this theory should regain supersymmetry and become equivalent to the standard maximally supersymmetric type-IIA string or M-theory in flat background.

4.2 General form of the solutions

Let us start with the $d = 11$ analog of (2.1), i.e. a $d = 11$ Minkowski spacetime with twists in $N = 1, 2, 3, 4$ orthogonal spatial planes and $y \equiv x_{11}$ ($n = 2N + 1, \ldots, 9$, $s = 1, \ldots, N$)

$$ds_{11}^2 = -dt^2 + dx_n^2 + dx_{11}^2 + \sum_{s=1}^{N} [dr_s^2 + r_s^2 (d\varphi_s + b_s dx_{11})^2] \,. \tag{4.1}$$

Being locally flat, this is an exact solution of 11-d supergravity and also of M-theory. Dimensional reduction in the x_{11} direction gives the following string-frame type-IIA metric, dilaton ϕ and R-R vector 1-form A [7, 16]

$$ds_{10A}^2 = f^{1/2} \left[-dt^2 + dx_n^2 + \sum_{s=1}^{N} (dr_s^2 + r_s^2 d\varphi_s^2) - f^{-1} \left(\sum_{s=1}^{N} b_s r_s^2 d\varphi_s \right)^2 \right] \tag{4.2}$$

$$e^{2(\phi - \phi_0)} = f^{3/2} \,, \qquad A = f^{-1} \sum_{s=1}^{N} b_s r_s^2 d\varphi_s \,, \qquad f = 1 + \sum_{s=1}^{N} b_s^2 r_s^2 \,. \tag{4.3}$$

This background may be interpreted as representing an orthogonal intersections of N individual R-R F7-branes [7, 9].

A supersymmetric solution preserving $1/2^{N-1}$ ($N > 1$) fraction of maximal supersymmetry (i.e. having 2^{6-N} real supersymmetries) is obtained by demanding[12]

$$b_1 = b_2 + \cdots + b_N \,.$$

This gives, in particular, a two-parameter family of three intersecting F7-branes, or a three-parameter family of four intersecting F7-branes.[13]

The simplest non-trivial case of (4.2), (4.3) is $N = 2$, or explicitly

$$\begin{aligned} ds_{10A}^2 &= f^{1/2} (-dt^2 + dx_n^2 + dr_1^2 + dr_2^2 + r_1^2 d\varphi_1^2 + r_2^2 d\varphi_2^2) - \\ &\quad - f^{-1/2} (b_1 r_1^2 d\varphi_1 + b_2 r_2^2 d\varphi_2)^2 \,, \end{aligned} \tag{4.4}$$

$$e^{2(\phi - \phi_0)} = f^{3/2} \,, \qquad A = f^{-1} (b_1 r_1^2 d\varphi_1 + b_2 r_2^2 d\varphi_2) \,, \qquad f = 1 + b_1^2 r_1^2 + b_2^2 r_2^2 \,. \tag{4.5}$$

[12] As already mentioned above, choices of other signs of b_s give equivalent solutions related by $\varphi_s \to -\varphi_s$.

[13] One may also consider the case of $N = 5$ with $x_{11} = y$ (and one of the angles playing the role of euclidean time). The corresponding type-IIA background may be interpreted as "F(-1) brane".

This 10-d background may be interpreted as describing an orthogonal intersection of the two R-R flux 7-branes (as expected for a localized intersection of two branes, this solution does not have radial symmetry in 4 common transverse directions). The 10-d description is valid for sufficiently small values of r_1, r_2 and $b_1 R, b_2 R$. This solution is related to the NS-NS type-IIA background (2.5) (or its T-dual (3.18),(3.19)) by U-duality, i.e. by the $T_9 S T_9$ sequence of duality transformations (which produces the "9-11 flip").

As discussed above, since the Killing spinor corresponding to (2.5) is constant in the $b_1 = b_2$ case, the supersymmetry is preserved by the duality, so the dual $b_1 = b_2$ R-R background is also 1/2 supersymmetric. Its explicit form is

$$\begin{aligned} ds^2 &= f^{1/2}\big(- dt^2 + dx_n^2 + dr_1^2 + dr_2^2 + r_1^2 d\varphi_1^2 + r_2^2 d\varphi_2^2 \big) - \\ &\quad - b^2 f^{-1/2} (r_1^2 d\varphi_1 + r_2^2 d\varphi_2)^2 \,, \qquad (4.6) \\ e^{2(\phi-\phi_0)} &= f^{3/2} \,, \qquad A = b f^{-1} (r_1^2 d\varphi_1 + r_2^2 d\varphi_2) \,, \qquad f = 1 + b^2 (r_1^2 + r_2^2) \,. \qquad (4.7) \end{aligned}$$

This solution is, in fact, equivalent to the supersymmetric "F5-brane" of [13]: the two backgrounds are related by the following coordinate transformation:

$$r_1 = r\cos\theta \,, \qquad r_2 = r \sin\theta \,, \qquad \varphi_1 = \tilde{\phi} + \psi \,, \qquad \varphi_2 = \tilde{\phi} - \psi \,. \qquad (4.8)$$

Since this solution has F_2 fluxes through two orthogonal planes (rather than F_4 flux) its more appropriate interpretation is that of two intersecting R-R F7-branes of equal fluxes.

By applying $T_9 S T_9$ duality to the 4-parameter background in (3.22) (or to its obvious $2N$ parameter generalization) one obtains more general fluxbrane intersections which, in addition to the R-R 1-form potential, have non-trivial NS-NS 2-form field.

4.3 Classical and quantum (non-perturbative) instabilities

Let us now discuss stability of non-supersymmetric F7-brane intersections, and possible fate of the unstable backgrounds. In order to make use of the discussion of the dual NS-NS model, let us assume that x_9 (one of the x_n coordinates in (4.2)) is periodic with radius R_9, so that the R-R (4.2), (4.3) and the NS-NS (2.1) backgrounds may be related through the "9-11" flip in 11 dimensions. The presence and nature of instabilities (classical or quantum) as well as the final state of evolution of an unstable background may depend on the values of the string coupling (or R_{11}) and the radius R_9. For example, the NS-NS background (2.5) is stable classically as a supergravity solution for large R_9, but for $R_9 < \sqrt{\alpha'}$ the theory is more appropriately represented by the T-dual background (3.18), (3.19), which is classically unstable (for certain values of the magnetic field parameters, see section 3.3) as a supergravity solution. In what follows, we will assume that the string coupling is small, i.e. $R_{11} \ll l_P$.

Let us first consider the issue of classical instabilities of the above R-R backgrounds at the supergravity level. A supergravity solution is stable if there are no tachyonic modes in the field equations expanded near the corresponding background. The NS-NS background (2.1) is stable at the supergravity level, in both $d = 10$ and $d = 11$ theories. In $d = 10$ this can be seen directly from the spectrum of the NS-NS string model in section 3 which contains no tachyons in the zero-winding sector, i.e. (as in the $N = 1$ case [6, 14]) there are no tachyonic states among the supergravity fluctuation modes for any value of the magnetic field. As a result, the F7-brane intersections that arise by the $11 \to 10$ dimensional reduction must also be stable (the 10-d supergravity modes are a subset of the 11-d supergravity modes, none of which is tachyonic).

Note that the fact that, e.g., the $N = 2$ R-R solution (4.4), (4.5) is U-dual to (2.5) (which is stable at the supergravity level) is not a priori sufficient to argue for the stability of the former background.[14] Instead, we use that since (2.5) is stable as a 10-d supergravity solution, its *direct* lift to 11 dimensions (4.1) should also be a stable 11-d supergravity background.[15] The R-R background is then a different 10-d reduction of the same stable 11-d background.

There is, however, a potential tachyonic instability of the flat 11-d space (4.1) with $b_1 \neq b_2 + \cdots + b_N$ at the full M-theory level. Indeed, we know that the subsector of M-theory representing the perturbative type-IIA strings in the 10-d background (2.1) is unstable in the winding string sector for certain values of the magnetic parameters. From the 11-d perspective, the instability is due to special winding membrane states. For example, in the $N = 2$ case such state is a counterpart of the winding string mode (3.15). Written in terms of 11-d parameters, its mass (3.16) is

$$M^2 = (4\pi^2 w R_9 R_{11} T_2)^2 - 8\pi^2 R_9 R_{11} T_2 (b_1 - b_2)\,, \tag{4.9}$$

where $T_2 = (2\pi l_P^3)^{-1}$ is the membrane tension. This means that type-IIA string theory in the R-R background (4.4), (4.5) should also be unstable at a non-perturbative level in a certain range of the parameters $b_1 - b_2$, R_9, R_{11} (eq. (4.9) is applicable for $R_9 \ll l_P$ or $R_{11} \ll l_P$).

In the case of the direct 11-d lift of the T-dual background (3.18), (3.19) the counterpart of the winding membrane state is a mode of the supergravity multiplet. In this case the presence of a tachyonic instability (and possible evolution of the unstable background) can be studied by using simply the 11-d supergravity equations (see also [14]).

[14] For example, (4.4) is U-dual also to the background (3.18) which, for a certain values of parameters, is unstable at the $d = 10$ supergravity level. Here the instability is developing in the compact isometry direction and thus is "overlooked" by the T-(or U-) duality argument (though $T-duality$ is a symmetry of the supergravity equations, it applies only in the presence of an isometry, while fluctuations may depend on the isometry direction).

[15] By the same logic, its reduction to 9 dimensions should be giving a stable 9-d solution. Indeed, it leads to the KK Melvin 9-d background which is known to be stable at the gravity level.

Let us now comment on possible quantum instabilities. The physical phenomenon leading to quantum decay of F7-brane configurations is the KK monopole (D6-brane) pair production in the magnetic field [30, 2, 5]. For the single magnetic field parameter $b_s = b$ ($N = 1$), the semiclassical instanton amplitude was discussed in [5, 11], and is of order e^{-I}, where $I \sim V_6 \kappa_{10}^{-2} R b^{-1}$. In the case of intersection of N F7-branes with generic parameters b_s [7] the production of monopoles should be at the expense of the magnetic energy density proportional to $\sum_s b_s^2$. It is natural to expect that due to pair creation the values of b_s should decrease (in general, at different rates), and the process should not stop until the background "rolls down" to one of the supersymmetric configurations with $\sum_s \pm b_s = 0$. When that happens, the instanton amplitude should vanish identically due to the presence of fermionic zero modes in the supersymmetric background.[16]

As was mentioned in section 3, which particular supersymmetric vacuum is reached should depend on initial values of the magnetic parameters b_s. For example, the initial configuration with $b_1 = b_2 + \epsilon$, $|\epsilon| \ll 1$ should evolve into the one with $b_1 = b_2$, while the configuration with $b_1 = \epsilon$, $b_2 \gg b_1$ may roll down to the trivial $b_1 = b_2 = 0$ vacuum.

5. Concluding remarks

We have seen that there is a novel type of R-R brane backgrounds preserving fractions of supersymmetries which are closely related (U-dual) to simple solvable NS-NS string models with continuous parameters. One interesting application of these backgrounds is to the study of the evolution of instabilities in closed string theory. As emphasized in the previous sections, in some related (T-dual, section 3.3) models the tachyons appear at the supergravity level, so the problem of decay of due to closed string tachyons may be effectively addressed by solving time-dependent supergravity equations of motion.[17] We have argued that non-supersymmetric, but tachyon-free, backgrounds should decay into stable supersymmetric ones with $\sum_{s=1}^{N} \pm b_s = 0$ (including, in particular, the trivial case of all $b_s = 0$).

There are also other potential applications which are worth of further study. The existence of the supersymmetric F1, F3, F5 branes representing orthogonal intersections of 4, 3 and 2 F7-branes which have, respectively, 4, 8 and 16 supersymmetries, may provide a new tool for "brane-world" model building. The dual NS-NS string models are also of interest in this context, being, in some respects, continuous-parameter analogs of non-compact orbifolds. The breaking of supersymmetry, and thus the splitting between fermion and boson masses in "parallel" dimen-

[16] Since the presence of supersymmetry for special values of b_s was not noticed in [7], it was assumed that these intersecting fluxbrane backgrounds should always decay non-perturbatively.

[17] Here the evolution is caused by unstable fluctuation modes in the supergravity multiplet itself. This is different from the case discussed in [21].

JHEP11(2001)065

sions here is controlled by a number of continuous parameters b_s. Putting standard D-branes in these NS-NS backgrounds may lead to new examples (in addition to branes on orbifolds and conifolds) of supergravity duals of non-conformal gauge theories with reduced amount of supersymmetry. We discuss some relevant solutions in appendix.

An interesting problem is to understand the structure of the world-volume theory, corresponding, in particular, to the most interesting case of the R-R F3-brane. While the standard supersymmetric p-branes (supported by delta-function sources that can be put anywhere in transverse space) are parametrized by harmonic functions decaying at infinity, have finite mass (or charge) density and can be BPS-superposed, this is not so for the above R-R Fp-branes. The latter are, in fact, more analogous to fractional Dp-branes or D$(p+q)$-branes wrapped on q-cycles which are supported by smooth fluxes instead of delta-functions and are localized in transverse space (cf. [31]; see also [16] for related discussion).

One may argue that the breaking of 3/4 of supersymmetry should imply the presence of 24 fermionic collective coordinates in the 1+3 dimensional world-volume theory. Since the F3-brane brane cannot move freely in the transverse 6-space (being the intersection of the three F7-branes it is pinned down at $r_1 = r_2 = r_3 = 0$) the translational symmetry breaking should not produce Goldstone bosons, but the breaking $SO(6) \to U(1)^3$ of the rotational symmetry in transverse directions should give $15 - 3 = 12$ massless scalar bosons on the brane. The expected $\mathcal{N} = \in$, $d = 4$ world-volume supersymmetry (corresponding to 8 preserved supercharges) can be realized by arranging these degrees of freedom into the 3 hypermultiplets (the numbers of on-shell modes do match: $\frac{1}{2} \times 24 = 12$), in agreement with a discrete symmetry interchanging the 3 planes.[18] The matching between bosons and fermions is also consistent with the expectation that there should be no additional vector gauge bosons on the F3-brane (the only non-trivial R-R flux is associated with the 1-form field having scalar gauge parameter).[19]

While the standard or fractional Dp-branes have "dual" open-string description in flat space (which gives an alternative way to determine the structure of massless modes on the brane) an existence of a similar description for the supersymmetric Fp-branes remains an open question. In general, it would be interesting to study the open superstring spectrum and possible D-brane configurations in the flat NS-NS backgrounds (2.1). For example, one can place a Dp-brane along parallel directions x_i. The corresponding open-string CFT is, in principle, straightforward to solve explicitly. The corresponding supergravity solution is also readily constructed and is discussed in appendix.

[18]One may use instead of (2.19) a more symmetric form of the supersymmetry condition $b_1 + b_2 + b_3 = 0$ to make this Z_3 symmetry manifest.

[19]Note also that there is no quantized number that could be associated with a rank of a non-abelian gauge group — the parameters b_s are continuous.

Acknowledgments

We are grateful to S. Frolov, G. Horowitz, I. Klebanov, H. Liu, R. Myers, H. Nieder, E. Poppitz and A. Uranga for useful discussions. J.R. was supported by Universidad de Buenos Aires and Conicet. The work of A.A.T. is partially supported by the DOE grant DE-FG02-91ER40690, PPARC SPG grant 00613, INTAS project 991590 and CRDF Award RPI-2108.

A. D3 branes with reduced supersymmetry

Here we construct D3 branes with reduced supersymmetry by adding magnetic fluxes in several planes of the transverse 6-space as in (2.1).[20]

As a first example, consider a D3 brane with transverse space $\mathrm{R}^2 \times \mathrm{R}^2 \times \mathrm{R}^2$. Suppose one of the 3 "parallel" coordinates $x_3 = y$ is compact with period $2\pi R$. Starting with the standard D3-brane solution and making formal coordinate redefinition $\varphi_s \to \varphi_s + b_s y$ which mixes y with angles in the three transverse planes, we get a new, globally-inequivalent, type-IIB solution with F_5 having the standard self-dual form, dilaton being constant, and the following metric

$$ds^2_{10B} = H^{-1/2}[-dt^2 + dx_1^2 + dx_2^2 + dy^2] + H^{1/2} \sum_{s=1}^{3} [dr_s^2 + r_s^2 (d\varphi_s + b_s dy)^2] \,. \quad \text{(A.1)}$$

Here $H \equiv h(r_1, r_2, r_3, \varphi_1 + b_1 y, \varphi_2 + b_2 y, \varphi_3 + b_3 y)$, where $h(r_1, r_2, r_3, \varphi_1, \varphi_2, \varphi_3)$ is the original harmonic function on the transverse R^6 space. For example, if one starts with the standard spherically-symmetric D3-brane, then

$$H = h = h_0 + \frac{L^4}{(r_1^2 + r_2^2 + r_2^2)^2} \,. \quad \text{(A.2)}$$

Alternatively, one may choose, for example, $H = h = \sum_{s=1}^{3} a_s \log \frac{r_s}{r_{0s}}$. If the theory in the absence of D3-brane ($H = 1$) is supersymmetric, i.e. b_s satisfy (2.19), then this solution preserves 1/8 of maximal type-IIB supersymmetry. Indeed, the conditions (2.20) viewed as restrictions on a 6-d spinor, i.e. $(I - \gamma_{4567})\epsilon_0 = 0$, $(I - \gamma_{4589})\epsilon_0 = 0$ should be supplemented by the D3-brane condition [32] $(I - i\gamma_{456789})\epsilon_0 = 0$.[21] These are equivalent to $\gamma_{45}\epsilon_0 = i\epsilon_0$, $\gamma_{67}\epsilon_0 = -i\epsilon_0$, $\gamma_{89}\epsilon_0 = -i\epsilon_0$, implying that 1/8 of maximal supersymmetry is preserved. This suggests that the corresponding world-volume 3-d gauge theory (assuming we compactify in y) should have four unbroken supercharges.

Consider the choice of $h_0 = 0$, i.e. $H = \frac{L^4}{(r_1^2 + r_2^2 + r_2^2)^2}$. In the UV region where all r_i are large the metric will factorize into AdS_5 with its S^1 direction y being "mixed"

[20] Solutions of this type in the $N = 1$ case where there is no supersymmetry were discussed in [17].

[21] Here we use the indices $4, \ldots, 9$ for the transverse directions, γ_k are 6-d Dirac matrices, and ϵ_0 is the 6-d part of the constant factor of the Killing spinor.

with φ_s coordinates of S^5.[22] One thus obtains a supergravity solution which should be dual to SYM theory with 0, 4, or 8 unbroken supersymmetries, depending on the values of b_s. The gauge theory (IR RG flow) interpretation of this background remains to be investigated.

A different D3-brane solution can be obtained by starting with the standard D2-brane solution with the transverse 7-space being flat "twisted" $S^1 \times \mathrm{R}^2 \times \mathrm{R}^2 \times \mathrm{R}^2$ space (a similar D4-brane solution is given explicitly below). If the corresponding harmonic function H is chosen to be φ_s-independent, then one may do T-duality in y (cf. (3.18), (3.19)), obtaining a D3-brane background which is similar to (A.1) at small r_s. In general, it will have an extra factor of f^{-1}, $f \equiv 1 + b_1^2 r_1^2 + b_2^2 r_2^2 + b_3^2 r_3^2$ in front of dy^2, with extra B_2 field and the dilaton $e^{2\phi - 2\phi_0} = f^{-1}$ as in (3.19). This is a particular case of the solutions obtained in [33].

One may also consider a D3-brane solution with the transverse 6-space being a product of a line R (or a circle) with the $N = 2$ twisted version of $\mathrm{R}^2 \times \mathrm{R}^2 \times S^1$. The corresponding $d = 4$ gauge theory on the brane will have $\mathcal{N}$=2 supersymmetry. Even though the dilaton here is constant, for a logarithmic choice of the corresponding harmonic function, this background may be reflecting (as in the orbifold examples in [34]) the running of the $\mathcal{N}$=2 gauge theory coupling with scale.

The T-dual to this D3-brane is a D4-brane solution with twisted $\mathrm{R}^2 \times \mathrm{R}^2 \times S^1$ as the 5-d transverse space. It has the form

$$
\begin{aligned}
ds^2_{10A} &= H^{-1/2}[-dt^2 + dx_i^2] + H^{1/2}\left[\sum_{s=1}^{2}(dr_s^2 + r_s^2(d\varphi_s + b_s dy)^2) + dy^2\right], \\
e^{2\phi} &= H^{-1/2}, \qquad F_4 = *(dH^{-1} \wedge dx_0 \wedge \cdots \wedge dx_4), \qquad \text{(A.3)}
\end{aligned}
$$

where $H = H$ is a harmonic function in R^4 with $\varphi_s \to \varphi_s + b_s y$, in particular, $H = H(r_1, r_2)$. For $b_1 = b_2$ the corresponding world-volume theory should have $\mathcal{N}$=1, $d = 5$ supersymmetry.

Analogous solutions can be also obtained by starting with the 11-d M5-brane background with the transverse 5-space $\mathrm{R}^2 \times \mathrm{R}^2 \times \mathrm{R}$ being mixed with a compact "parallel" coordinate y ($i = 1, \dots, 4$)

$$
\begin{aligned}
ds^2_{11} &= H^{-1/3}\Big[-dt^2 + dx_i^2 + dy^2\Big] + \\
&+ H^{2/3}\Big[dr_1^2 + r_1^2(d\varphi_1 + b_1 dy)^2 + dr_2^2 + r_2^2(d\varphi_2 + b_2 dy)^2 + dz^2\Big]. \qquad \text{(A.4)}
\end{aligned}
$$

Here $H = H(r_1, r_2)$ is again the corresponding harmonic function. Dimensional reduction in y gives a generalized type-IIA D4-brane solution with a R-R F_2 flux in

[22] We are grateful to I. Klebanov for suggesting this possibility and pointing out a relation to the work of [33].

the two transverse planes

$$
\begin{aligned}
ds^2_{10A} = f^{1/2}\Big(H^{-1/2}[-dt^2+dx_i^2] + H^{1/2}[dr_1^2 + r_1^2 d\varphi_1^2 + dr_2^2 + r_2^2 d\varphi_2^2 + dz^2 - \\
- Hf^{-1}(b_1 r_1^2 d\varphi_1 + b_2 r_2^2 d\varphi_2)^2]\Big)\,, \qquad \text{(A.5)} \\
e^{2\phi} = f^{3/2} H^{-1/2}\,, \quad A_1 = Hf^{-1}(b_1 r_1^2 d\varphi_1 + b_2 r_2^2 d\varphi_2)\,, \quad f = 1 + H(b_1^2 r_1^2 + b_2^2 r_2^2)\,.
\end{aligned}
$$

By T-duality in x_4, we can also get another D3-brane solution with dilaton $e^{2\phi} = f(r_1, r_2)$.

Dimensional reduction of (A.4) along x_4 gives a D4 brane solution of type-IIA supergravity with the metric

$$
\begin{aligned}
ds^2_{10A} = H^{-1/2}\Big[-dt^2 + dx_i^2 + dy^2\Big] + \\
+ H^{1/2}\Big[dr_1^2 + r_1^2(d\varphi_1 + b_1 dy)^2 + dr_2^2 + r_2^2(d\varphi_2 + b_2 dy)^2 + dz^2\Big]\,, \qquad \text{(A.6)}
\end{aligned}
$$

and the dilaton $e^{2\phi} = H^{-1/2}$. For the harmonic function being φ_s and z independent, e.g., $H = \frac{L^2}{r_1^2 + r_2^2}$, this solution should provide a dual description to SYM in 3+1 dimensions, with the number of unbroken supersymmetries $\mathcal{N} = 0$ for generic b_s, or $\mathcal{N} = 2$ for $b_1 = b_2$.

It would be interesting to study further the low-energy gauge theories corresponding to these solutions (see in this connection [33]).

References

[1] G.W. Gibbons, *Quantized flux tubes in Einstein-Maxwell theory and noncompact internal spaces*, in *Fields and geometry*, A. Jadczyk ed., World Scientific, 1986;
G.W. Gibbons and K.-I. Maeda, *Black holes and membranes in higher dimensional theories with dilaton fields*, *Nucl. Phys.* **B 298** (1988) 741.

[2] F. Dowker, J.P. Gauntlett, D.A. Kastor and J. Traschen, *Pair creation of dilaton black holes*, *Phys. Rev.* **D 49** (1994) 2909 [hep-th/9309075];
F. Dowker, J.P. Gauntlett, S.B. Giddings and G.T. Horowitz, *On pair creation of extremal black holes and Kaluza-Klein monopoles*, *Phys. Rev.* **D 50** (1994) 2662 [hep-th/9312172].

[3] A.A. Tseytlin, *Melvin solution in string theory*, *Phys. Lett.* **B 346** (1995) 55 [hep-th/9411198].

[4] J.G. Russo and A.A. Tseytlin, *Exactly solvable string models of curved space-time backgrounds*, *Nucl. Phys.* **B 449** (1995) 91 [hep-th/9502038].

[5] F. Dowker, J.P. Gauntlett, G.W. Gibbons and G.T. Horowitz, *The decay of magnetic fields in Kaluza-Klein theory*, *Phys. Rev.* **D 52** (1995) 6929 [hep-th/9507143].

JHEP11(2001)065

[6] J.G. Russo and A.A. Tseytlin, *Magnetic flux tube models in superstring theory*, *Nucl. Phys.* **B 461** (1996) 131 [hep-th/9508068].

[7] F. Dowker, J.P. Gauntlett, G.W. Gibbons and G.T. Horowitz, *Nucleation of p-branes and fundamental strings*, *Phys. Rev.* **D 53** (1996) 7115 [hep-th/9512154].

[8] R. Emparan, *Composite black holes in external fields*, *Nucl. Phys.* **B 490** (1997) 365 [hep-th/9610170].

[9] J.G. Russo and A.A. Tseytlin, *Green-Schwarz superstring action in a curved magnetic Ramond-Ramond background*, *J. High Energy Phys.* **04** (1998) 014 [hep-th/9804076].

[10] C.-M. Chen, D.V. Gal'tsov and S.A. Sharakin, *Intersecting M-fluxbranes*, *Grav. Cosmol.* **5** (1999) 45 [hep-th/9908132].

[11] M.S. Costa and M. Gutperle, *The Kaluza-Klein Melvin solution in M-theory*, *J. High Energy Phys.* **03** (2001) 027 [hep-th/0012072].

[12] P.M. Saffin, *Gravitating fluxbranes*, *Phys. Rev.* **D 64** (2001) 024014 [gr-qc/0104014].

[13] M. Gutperle and A. Strominger, *Fluxbranes in string theory*, *J. High Energy Phys.* **06** (2001) 035 [hep-th/0104136].

[14] J.G. Russo and A.A. Tseytlin, *Magnetic backgrounds and tachyonic instabilities in closed superstring theory and M-theory*, *Nucl. Phys.* **B 611** (2001) 93 [hep-th/0104238].

[15] M.S. Costa, C.A.R. Herdeiro and L. Cornalba, *Flux-branes and the dielectric effect in string theory*, hep-th/0105023;
R. Emparan, *Tubular branes in fluxbranes*, *Nucl. Phys.* **B 610** (2001) 169 [hep-th/0105062];
J.F. Sparks, *Kaluza-Klein branes*, hep-th/0105209;
D. Brecher and P.M. Saffin, *A note on the supergravity description of dielectric branes*, *Nucl. Phys.* **B 613** (2001) 218 [hep-th/0106206].

[16] A.M. Uranga, *Wrapped fluxbranes*, hep-th/0108196.

[17] A.A. Tseytlin, *Magnetic backgrounds and tachyonic instabilities in closed string theory*, hep-th/0108140.

[18] Y. Sekino and T. Yoneya, *From supermembrane to matrix string*, hep-th/0108176;
L. Motl, *Melvin matrix models*, hep-th/0107002.

[19] T. Suyama, *Properties of string theory on Kaluza-Klein Melvin background*, hep-th/0110077.

[20] T. Takayanagi and T. Uesugi, *Orbifolds as Melvin geometry*, hep-th/0110099.

[21] A. Adams, J. Polchinski and E. Silverstein, *Don't panic! closed string tachyons in ALE space-times*, *J. High Energy Phys.* **10** (2001) 029 [hep-th/0108075].

[22] I. Chepelev and A.A. Tseytlin, *Long-distance interactions of branes: correspondence between supergravity and super Yang-Mills descriptions*, *Nucl. Phys.* **B 515** (1998) 73 [hep-th/9709087].

[23] M.J. Duff, H. Lu and C.N. Pope, *Supersymmetry without supersymmetry*, *Phys. Lett.* **B 409** (1997) 136 [hep-th/9704186].

[24] A. Dabholkar, *Strings on a cone and black hole entropy*, *Nucl. Phys.* **B 439** (1995) 650 [hep-th/9408098];
D.A. Lowe and A. Strominger, *Strings near a Rindler or black hole horizon*, *Phys. Rev.* **D 51** (1995) 1793 [hep-th/9410215].

[25] R. Rohm, *Spontaneous supersymmetry breaking in supersymmetric string theories*, *Nucl. Phys.* **B 237** (1984) 553.

[26] C. Kounnas and B. Rostand, *Coordinate dependent compactifications and discrete symmetries*, *Nucl. Phys.* **B 341** (1990) 641.

[27] R. Gregory and R. Laflamme, *Black strings and p-branes are unstable*, *Phys. Rev. Lett.* **70** (1993) 2837 [hep-th/9301052].

[28] G.T. Horowitz and K. Maeda, *Fate of the black string instability*, *Phys. Rev. Lett.* **87** (2001) 131301 [hep-th/0105111].

[29] J. Figueroa-O'Farrill and G. Papadopoulos, *Homogeneous fluxes, branes and a maximally supersymmetric solution of M-theory*, *J. High Energy Phys.* **08** (2001) 036 [hep-th/0105308].

[30] E. Witten, *Instability of the Kaluza-Klein vacuum*, *Nucl. Phys.* **B 195** (1982) 481.

[31] I.R. Klebanov and A.A. Tseytlin, *Gravity duals of supersymmetric* $\mathrm{SU}(N)\times\mathrm{SU}(N+M)$ *gauge theories*, *Nucl. Phys.* **B 578** (2000) 123 [hep-th/0002159];
J.M. Maldacena and C. Nuñez, *Towards the large-N limit of pure* $N=1$ *super Yang-Mills*, *Phys. Rev. Lett.* **86** (2001) 588 [hep-th/0008001].

[32] M.J. Duff and J.X. Lu, *The selfdual type-IIB superthreebrane*, *Phys. Lett.* **B 273** (1991) 409.

[33] A. Bergman, K. Dasgupta, O.J. Ganor, J.L. Karczmarek and G. Rajesh, *Nonlocal field theories and their gravity duals*, hep-th/0103090.

[34] I.R. Klebanov and N.A. Nekrasov, *Gravity duals of fractional branes and logarithmic RG flow*, *Nucl. Phys.* **B 574** (2000) 263 [hep-th/9911096];
M. Bertolini, P. Di Vecchia, M. Frau, A. Lerda, R. Marotta and I. Pesando, *Fractional D-branes and their gauge duals*, *J. High Energy Phys.* **02** (2001) 014 [hep-th/0011077];
J. Polchinski, $N=2$ *gauge-gravity duals*, *Int. J. Mod. Phys.* **A 16** (2001) 707 [hep-th/0011193].